Synchronization and information processing by an on-off coupling

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(Received 4 October 2001; revised manuscript received 2 January 2002; published 6 May 2002)

This paper proposes an on-off coupling process for chaos synchronization and information processing. An in depth analysis for the net effect of a conventional coupling is performed. The stability of the process is studied. We show that the proposed controlled coupling process can locally minimize the smoothness and the fidelity of dynamical data. A digital filter expression for the on-off coupling process is derived and a connection is made to the Hanning filter. The utility and robustness of the proposed approach is demonstrated by chaos synchronization in Duffing oscillators, the spatiotemporal synchronization of noisy nonlinear oscillators, the estimation of the trend of a time series, and restoration of the contaminated solution of the nonlinear Schrödinger equation.

DOI: 10.1103/PhysRevE.65.056210 PACS number(s): 05.45.Gg, 05.45.Tp, 05.45.Xt

I. INTRODUCTION

The topic of synchronization and chaos control has attracted much attention in the past decade [1–10]. Active research in this area has contributed greatly to the understanding of a wide class of complex phenomena, including synchronization in secure communication [2], electronic circuits [3], and nonlinear optics [4], coherence transfer in magnetic resonance [5], and oscillation in chemical and biological systems [6]. In fact, the study of this topic leads to many practical applications in the aforementioned fields. Many useful synchronization approaches were proposed for shock capturing [7], turbulence control [8], and signal processing [9]. A key element used in many of these studies is the direct, simultaneous coupling of dynamical variables [9], see Eq. (1). Obviously, it will be interesting to study how an alternative coupling scheme affects chaos synchronization and dynamical behavior of nonlinear systems. In particular, it is generally unknown if an indirect, on-off coupling scheme, which couples the temporal values of dynamical variables, can be used to achieve chaos synchronization. Moreover, it is beneficial to explore further the potential of synchronization and chaos control techniques for digital signal processing (DSP) and information autoregression (IAR). Both DSP and IAR are of crucial importance to telecommunication, biomedical imaging, pattern recognition, missile guidance, target tracking, autonomous control, etc. The main objective of this paper is to explore effectiveness of an indirect, on-off coupling process for the chaos synchronization. We are also interested in the use of this synchronization-based technique for DSP and IAR.

This paper is organized as follows. The coupling process is presented in Sec. II. Theoretical analysis of the process is given in Sec. III. Analyses of stability conditions, information processing and filter properties are carried out. Such analyses provide a guide to the selection of coupling parameters. Section IV is devoted to numerical experiments. The utility and effectiveness of the proposed coupling process for chaos synchronization, signal processing, and data regression are demonstrated. This paper ends with a conclusion.

II. THE ON-OFF COUPLING PROCESS

We consider a nonlinear dynamical system consisting of N identical subsystems, whose dynamical variables are directly coupled via the nearest and next-to-the-nearest neighborhood sites

$$\frac{du_j}{dt}=f(u_j)+N_j(t)+a(u_{j+2}-u_{j+1})+(b-3a)(u_{j+1}-u_j)$$
$$+a(u_{j-2}-u_{j-1})+(b-3a)(u_{j-1}-u_j), \qquad (1)$$

where $u_j\in[0,\infty)\times R^n$, f is a nonlinear function of u_j, which might undergo chaotic dynamics, $N_j(t)$ is the noise, a and b are scalar hyperdiffusive and diffusive coupling parameters, respectively. The introduction of the hyperdiffusive coupling is a minor generalization of the coupling process given by Lindner *et al.* [9] and could make the coupling slightly more effective. The coupling scheme in Eq. (1) is strongly dissipative and a synchronous state can be attained for the chaotic Duffing oscillators by using appropriate coupling parameters [10]. We refer to this coupling as direct and simultaneous in the sense that it directly connects the dynamical variables of many subsystems during the dynamical process governed by the nonlinear relation $f(u_j)$. As couplings are fundamental to the behavior of dynamical systems, it is important and interesting to study the effect of different coupling strategies. In this work, we propose the following indirect, on-off coupling process:

$$\frac{du_j}{dt}=\theta_{t_1,t_2,\ldots,t_k}(t)[f(u_j)+N_j(t)]+\overline{\theta}_{t_1,t_2,\ldots,t_k}(t)$$
$$\times[a(u_{j+2}-u_{j+1})+(b-3a)(u_{j+1}-u_j)$$
$$+a(u_{j-2}-u_{j-1})+(b-3a)(u_{j-1}-u_j)], \qquad (2)$$

where $\theta_{t_1,t_2,\ldots,t_k}(t)$ is a control function that consists of a periodic train of Heaviside type intervals and $\overline{\theta}_{t_1,t_2,\ldots,t_k}(t)$ is the complement of $\theta_{t_1,t_2,\ldots,t_k}(t)$ in the domain $[0,\infty)$ [i.e., $\overline{\theta}_{t_1,t_2,\ldots,t_k}(t)=1-\theta_{t_1,t_2,\ldots,t_k}(t)$]. Both control functions are depicted in Fig. 1. During the time interval $0\le t$

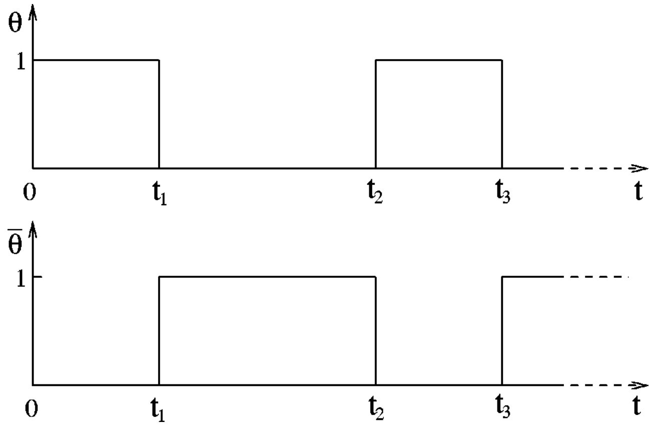

FIG. 1. The control functions.

$\leqslant t_1$, each subsystem, together with possible noise, evolves freely without coupling. At time t_1, the coupling is switched on and the time evolution of the dynamical variables of many subsystems is entirely governed by the proposed controlled coupling until t_2. These on-off coupling processes are periodically arrayed in the time domain of $[0,\infty)$. In the rest of this paper, we offer a theoretical analysis of the coupling process and demonstrate its use for chaos synchronization and DSP.

III. THEORETICAL ANALYSES

In this section, we first examine the stability condition for the time integration of the on-off coupling process. We then investigate the use of the proposed process for the trend estimation of time series. Finally, we study the filter property of the process.

A. Stability conditions

In the conventional coupling, Eq. (1), it is the dynamical variables that are coupled, whereas in the on-off coupling, Eq. (2), it is the temporal results (values) of dynamical variables that are coupled and treated for a while before the onset of the next dynamical evolution. A key characteristic of the on-off process is that the coupling and the nonlinear evolution will never coexist during whole process, so that the detailed analysis of the *net effect* of the coupling in Eq. (1) becomes possible. At time $t_1 \leqslant t \leqslant t_2$, a discretization of the isolated system Eq. (2), written in terms of a (time) iteration form, can be given by

$$u_j^{s+1}=u_j^s+R(u_{j-1}^s-2u_j^s+u_{j+1}^s)+T(u_{j-2}^s-4u_{j-1}^s+6u_j^s-4u_{j+1}^s+u_{j+2}^s), \tag{3}$$

$$u_j^0=u_j(t_1), \quad j=1,\ldots,N, \text{ and } s=0,1,\ldots,S,$$

where

$$R=b\Delta t, \quad T=a\Delta t \tag{4}$$

and iteration parameter

$$S=(t_2-t_1)/\Delta t. \tag{5}$$

It should be noted that T and R are actually scaled hyperdiffusive and diffusive parameters, respectively. The controlled coupling process (3) is conditionally stable, similar to the normal coupling system. It is known that the stability of the normal coupling system is governed by the stability of both the intrinsic dynamics and the coupling terms [10]. For the on-off process, the dynamical evolution is linear when the coupling is acting, so that the stability analysis can be relatively easily carried out. Neglecting the boundary modifications, we can rewrite Eq. (3) in a matrix form,

$$U^{s+1}=AU^s, \tag{6}$$

where

$$U^s=(u_1^s,u_2^s,\ldots,u_N^s)^T, \tag{7}$$

and the pentadiagonal matrix A has nonzero coefficients:

$$a_{j,j-2}=a_{j,j+2}=T, \tag{8}$$

$$a_{j,j-1}=a_{j,j+1}=R-4T, \tag{9}$$

$$a_{j,j}=1-2R+6T, \tag{10}$$

for $j=1,2,\ldots,N$. If all of the eigenvalues of A are smaller than unity, the iterative correction

$$\epsilon^{s+1}=||U^{s+1}-U^s|| \tag{11}$$

will decay, then the process is stable with respect to a large number of iterations (or long-time evolution). Since each diagonal term of the matrix is a constant, the eigenvectors of A can be represented in terms of a complex exponential form,

$$U_j^s=q^s e^{i\gamma j}, \tag{12}$$

where $i=\sqrt{-1}$ and γ is a wave number that can be chosen arbitrarily. Substituting Eq. (12) into Eq. (6) and removing the common term $e^{i\gamma j}$, we obtain an explicit expression for the eigenvalue q:

$$q=1+2T(2\cos^2\gamma-\cos\gamma-1)+2(R-3T)(\cos\gamma-1). \tag{13}$$

For a stable process, the magnitude of this quantity is required to be smaller than unity,

$$q^2<1. \tag{14}$$

For the case of $T=0$, q is the maximum when $\cos\gamma=-1$. Thus, the controlled coupling process is stable provided $0<R<\frac{1}{2}$. On the other hand, if $R=0$, our analysis indicates that $0>T>-\frac{1}{2}1/(\cos^2\gamma-2\cos\gamma+1)$, which is also derived by taking an extremum of q at $\cos\gamma=-1$. Therefore, the controlled coupling process is stable provided $0>T>-\frac{1}{8}$. Under these conditions, we have $\epsilon^{s+1}\leqslant\epsilon^s$, for any $s\in\mathbb{Z}^+$.

Although the present stability analysis is limited to the processing of the dynamical variables, the results of the analysis can be used for guiding the parameter selection of the conventional coupling in Eq. (1). The stability conditions of the conventional coupled process has not been fully clarified and the choice of the coupling parameter is essentially empirical. For such a process, the stability constraints on the coupling parameters should be within the neighborhood of the results obtained for the on-off coupling.

It is interesting to note that the controlled coupling process, Eq. (2), provides not only an approach for the investigation of nonlinear dynamical systems, but also a powerful, realistic algorithm for DSP and IAR. We will next explore the DSP and IAR properties of the on-off coupling process. These properties actually in return enhance the comprehensibility of the effect of the coupling terms to the nonlinear dynamical system, in particular, provide a DSP viewpoint on why the coupling can induce synchronization.

B. Information processing

For IAR, the on-off coupling forms an interesting approach for the trend estimation problem of a time series [11], which is useful for data interpretation and long-term forecasting. The primary goal of trend estimation is to extract trend component from time series, which is usually assumed to be the sum of the trend, the seasonal component, and random noise [11]. Nonparametric style methods are widely used for trend estimation. Nonparametric approaches make no assumption about the trend function, and allow great flexibility in the possible form of the fitting curve [12]. One common feature of all nonparametric approaches is that there are one or more smoothing parameters that govern the fundamental tradeoff between the bias and the variance of estimates, as well as tradeoff between the smoothness and the fidelity (closeness to time series) of estimated trend [12].

What is more relevant to the on-off coupling process is a class of nonparametric trend estimation methods that were constructed by explicitly quantifying the global competition between the two conflicting features: the smoothness and the fidelity. The earliest motivation to this approach dates back to 1923 when Whittaker [13] introduced *graduation*. The fidelity and smoothness are defined as the sum of squares of the residual and the accumulated power of the finite difference, respectively. Hodrick and Prescott [14] provided a concrete version of Whittaker's approach, which has been extensively used in the real business cycle literature for detrending. Recently, Mosheiov and Raveh [15] proposed a linear programing method to estimate the trend by employing the sum of the *absolute* values rather than the common sum of squares to measure the smoothness and fidelity. In the present approach, the terms in the first and second brackets of Eq. (3) are the second order and fourth order pointwise measures of smoothness, which are denoted as $\Delta^2 u_j^s$ and $\Delta^4 u_j^s$, respectively. To have a better understanding of this controlled coupling process, we rewrite Eq. (3) as

$$[u_j(t_1)-u_j^s]+[Rv_j^{s-1}+Tw_j^{s-1}]=0, \quad j=1,\ldots,N, \tag{15}$$

where

$$v_j^{s-1}=\sum_{k=0}^{s-1}\Delta^2 u_j^k,$$

and

$$w_j^{s-1}=\sum_{k=0}^{s-1}\Delta^4 u_j^k.$$

It is clear that the expression in the first square bracket is the local measure of the fidelity, while the expression in the second square bracket is the accumulative local measures of smoothness. They are local measures, since calculation involved in Eq. (15) is compactly supported. At each step of the iteration, this process guarantees that the sum of the local deviation from $u_j(t_1)$ and the accumulative local measure of smoothness equals zero. As such, the result of each iteration is optimal in the sense of minimization, for the given input and the set of parameters R and T. In comparison, the previous IAR methods [13–15] seek for global minimizations over the entire domain to obtain optimal estimates, while the present controlled coupling process forces the sum of smoothness and fidelity to pass through zero at each iteration to give an optimal trend.

Besides the minimization of two properties, another important aspect of the construction of nonparametric trend estimation method is the tradeoff balance. Since the previously discussed iteration correction

$$\epsilon^{s+1}=||U^{s+1}-U^s||=||R\Delta^2 U^s+T\Delta^4 U^s|| \tag{16}$$

is actually a global smoothness measure of estimated trend at the sth iteration, one can argue that as the iterative process lasts longer, the estimated trend becomes smoother, while the deviation of U^s from $U^0=U(t_1)=[u_1(t_1),u_2(t_1),\ldots,u_N(t_1)]^T$ becomes larger. In other words, the tradeoff between the smoothness and the fidelity in the coupling terms is governed by the iteration parameter S, as well as scaled hyperdiffusive and diffusive parameters T and R. Therefore, the on-off coupling process forms a useful approach to IAR applications. The advantage of the proposed controlled coupling lies in its localization and simplicity.

For a real application of IAR, the choice of smoothing parameters is clearly a crucial issue. Fortunately, this problem has been extensively studied in the literature, since every nonparametric estimation approach encounters the same. The current widely used criteria for choosing smoothing parameters includes cross validation and generalized cross validation [12]. These methods are also applicable within the on-off coupling process.

The IAR properties of the coupling can be understood from the point of view of nonlinear dynamical studies. For a coupled nonlinear dynamics, the IAR smoothness property is actually the similarity of subsystems, while the fidelity is the deviation of dynamical system subjected to coupling. A synchronous state is a state having perfect similarity, and the largest deviation. It is obvious that the convergence time of a synchronous state can be controlled through coupling strengths a and b for the general coupling (1). Since T and R

TABLE I. The filter weights $[W(k,6)]$ of the controlled coupling process $(T=0)$.

k		$R=0.4$	$R=0.1$
0	$924R^6-1512R^5+1050R^4-400R^3+90R^2-12R+1$	0.181824	0.390804
1	$-792R^6+1260R^5-840R^4+300R^3-60R^2+6R$	0.154368	0.227808
2	$495R^6-720R^5+420R^4-120R^3+15R^2$	0.12672	0.065295
3	$-220R^6+270R^5-120R^4+20R^3$	0.07168	0.01048
4	$66R^6-60R^5+15R^4$	0.039936	0.000966
5	$-12R^6+6R^5$	0.012288	0.000048
6	R^6	0.004096	0.000001

are scaled coupling strengths, this essentially agrees with the previous IAR investigation for fixed S. On the other hand, the local minimization property of the on-off coupling suggests that the coupling in nonlinear dynamics also should have some optimization properties, which, however, are not clear at the present stage, and deserve to be explored further.

C. Filter properties

For DSP, it is important to analyze the relationship between the proposed process and digital filters. To this end, we explore a weighted average representation of the controlled coupling process. For simplicity, we consider the case of $T=0$. We first set S to 1, i.e., $t_2-t_1=\Delta t$, then the controlled coupling process gives

$$u_j^1=Ru_{j-1}(t_1)+(1-2R)u_j(t_1)+Ru_{j+1}(t_1),\quad j=1,\ldots,N, \tag{17}$$

which is clearly a local weighted average form for $u_j(t_1)$. In general, after S iterations, the controlled coupling process can be represented as:

$$u_j^S=\sum_{k=j-S}^{j+S}W(k,S)u_k(t_1), \tag{18}$$

where weight function $W(k,S)$ has the general form,

$$W(k,S)=\begin{cases}\sum_{h=0}^{(S-k)/2}\,, & g(k,S,2h) \quad \text{when } S-k \text{ even},\\ \sum_{h=1}^{(S-k+1)/2}\,, & g(k,S,2h-1) \quad \text{when } S-k \text{ odd},\end{cases} \tag{19}$$

and

$$g(k,S,h)=\frac{S!R^{S-h}(1-2R)^h}{\left(\frac{S+k-h}{2}\right)!\left(\frac{S-k-h}{2}\right)!h!}. \tag{20}$$

It can be easily verified that,

$$\sum_{k=j-S}^{j+S}W(k,S)=1, \tag{21}$$

and

$$W(-k,S)=W(k,S),\quad \forall k=1,\ldots,S. \tag{22}$$

Equation (18) indicates that the controlled coupling process can be viewed as a kernel smoother for IAR [12] and a low-pass filter for DSP [16]. Two estimators in IAR and DSP are essentially equivalent in the present content. As a popular trend estimation method, a kernel smoother provides a good estimate to trend, which is actually a low-frequency component of time series. In general, the word filter suggests that estimation will select a band of frequencies. A low-pass filter has a pass band and a stop band at low- and high-frequency parts, respectively. Consequently, the low-frequency response of time series can pass through low-pass filtering, while the high-frequency response will be attenuated. Therefore, the output of low-pass filtering is obviously a good estimate to trend.

In terms of assigning the weights, the controlled coupling process filter is also analogous to those of other kernel regression methods. For a reasonable choice of R and S, the greater or smaller weight will be assigned to the points close to or far away from $u_j(t_1)$, respectively, see Table I and Fig. 2. Obviously, the distribution of the weights has a Gaussian shape when S is sufficiently large. It is noted that the implementation of the controlled coupling process becomes very simple owing to the existence of Eq. (18). The S steps iteration can be performed through one step filtering. Numerically, the weighted average form (18) is very useful. In com-

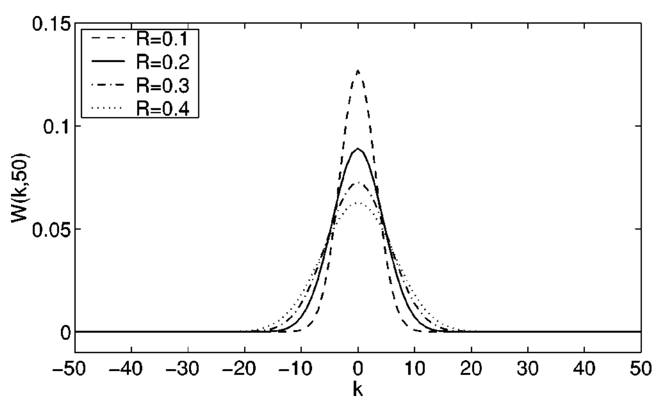

FIG. 2. The filter weights of the controlled coupling process $(T=0)$.

parison to the previous IAR methods [13–15], the implementation of the on-off coupling process is quite simple.

A simple moving average filter can be constructed by convolving the mask $(\frac{1}{2},\frac{1}{2})$ with itself, $2S$ times. When $S=1$, such a filter is the Hanning filter [17] $(\frac{1}{4},\frac{1}{2},\frac{1}{4})$. In our case, if we set $R=\frac{1}{4}$ in the Eq. (17), the present controlled coupling process has the same filter coefficients as those of the Hanning filter. Thus, the proposed controlled coupling process filter can be viewed as a generalization of the Hanning filter.

We can also investigate the meaning of the DSP property of the coupling for the nonlinear dynamics. In the sense of DSP, the coupling process is a kind of low-pass filtering. The difference among subsystems constitutes the high-frequency component, and will be eliminated by the (low-pass filter) coupling during the time evolution, while the similarity among subsystems represents the low-frequency component, and will be enhanced by the coupling. As such, synchronous state will be attained eventually. It is noted that although our conclusion is drawn for the coupling in the linear evolution, the same is true when nonlinear evolution is involved. Because the low-pass filter property of the coupling term does not change with the nonlinearity.

IV. NUMERICAL EXAMPLES

In the rest of this paper, we demonstrate the utility of the proposed approach for chaos synchronization, trend estimation, and noise reduction. To this end, a few numerical experiments are carried out.

A. Application to chaos synchronization

First, we consider chaos synchronization in Duffing oscillators with periodic boundary [10]

$$\dot{u}_j=(\dot{x}_j,\dot{y}_j)=[y_j,-\alpha y_j-x_j^3+E\cos(\omega t)], \qquad (23)$$

where $(\alpha,E,\omega)=(0.3,11.0,1.0)$ and $j=1,2,\ldots,6$. Both the controlled coupling (2) and the normal coupling $[\epsilon(y_{j+1}-y_j)+\epsilon(y_{j-1}-y_j)]$ [10] are employed to synchronize the spatially extended nonlinear system (23). It is noted that the diffusive parameter ϵ is actually b in Eq. (1) in our nomenclature, and the normal coupling is equivalent to the coupling in Eq. (1) with $a=0$ (i.e., $T=a\Delta t=0$). In numerical study, the same diffusive parameter will be employed for both coupling schemes, i.e., $\epsilon=0.5/\Delta t$ and $R=0.5$, and we choose $T=0$ in the proposed coupling. The on-off switching is fixed as $t_{2k+1}-t_{2k}=10\Delta t$ and $t_{2(k+1)}-t_{2k+1}=S\Delta t=10\Delta t$, for $k=0,1,2,\ldots$. A synchronous state can be obtained by using the on-off coupling. The average absolute differences of coupled oscillators are depicted in Fig. 3. It is seen that by using the controlled coupling, the synchronous state is actually reached much more quickly than by using the normal coupling.

Next, we consider a spatiotemporal synchronization of noisy nonlinear oscillators [9]

$$\dot{x}_j=Kx_j-K'x_j^3+A\sin(2\pi ft)+N_j(t), \qquad (24)$$

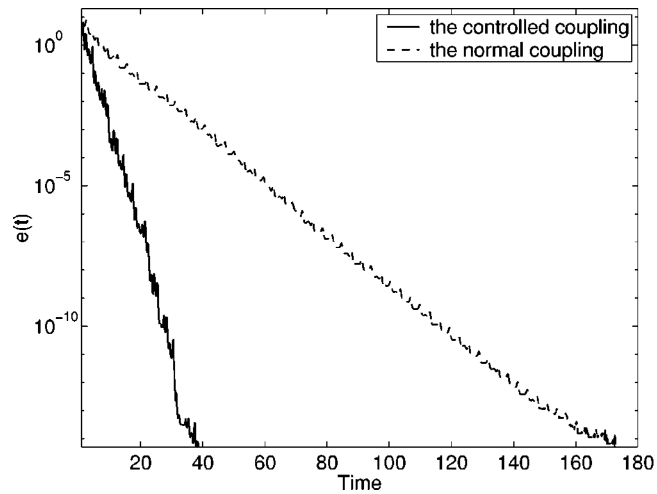

FIG. 3. Plot of the absolute difference, $e(t)=\frac{1}{12}\Sigma_{i=1}^{6}(|x_i-x_{i-1}|+|y_i-y_{i-1}|)$, of coupled Duffing oscillators.

where $(K,K',A,f)=(2.1078,1.4706,1.3039,0.1162)$, $j=1,2,\ldots,101$, and the Gaussian white noise $N_j(t)$ has zero mean and variance σ^2. Both the controlled coupling (2) and the normal coupling $[\epsilon(x_{j+1}-x_j)+\epsilon(x_{j-1}-x_j)]$ [9] are implemented for Eq. (24) with periodic boundary conditions. In the proposed coupling, we set $T=0$, $S=20$, and $t_{2k+1}-t_{2k}=20\Delta t$. The spatiotemporal order is characterized by a signal-to-noise ratio (SNR),

$$\Omega_{\mathrm{SNR}}=10\log_{10}\left[\frac{\text{original signal power}}{\text{estimated noise power}}\right], \qquad (25)$$

where the original signal is the sampled response of the nonlinear system (24) without the white noise, while the estimated noise is the difference between the noisy response and the original signal. By fixing Δt at 0.001, the SNR enhancement, defined as the increase in SNR owing to couplings, is displayed in Fig. 4. It is seen that both couplings give excellent enhancement over a wide range of noise power and coupling strength, while the proposed coupling has a larger maximum improvement.

Two SNR enhancement patterns shown in Fig. 4 only have some differences in the line $R=\epsilon\Delta t=0.5$. When $R>0.5$, the on-off coupling is unstable, SNR enhancement quickly decays to zero. This exactly agrees with the previous

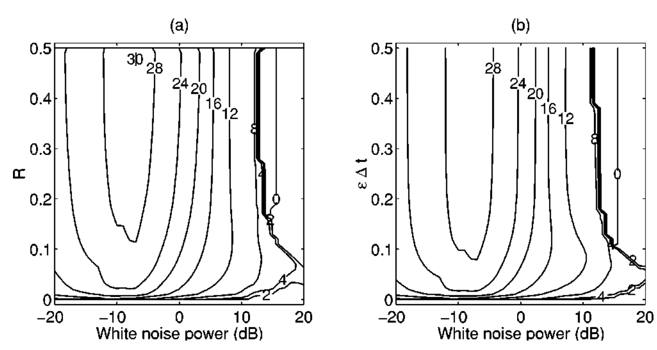

FIG. 4. The SNR enhancement of noisy nonlinear oscillators. (a) The controlled coupling; (b) the normal coupling.

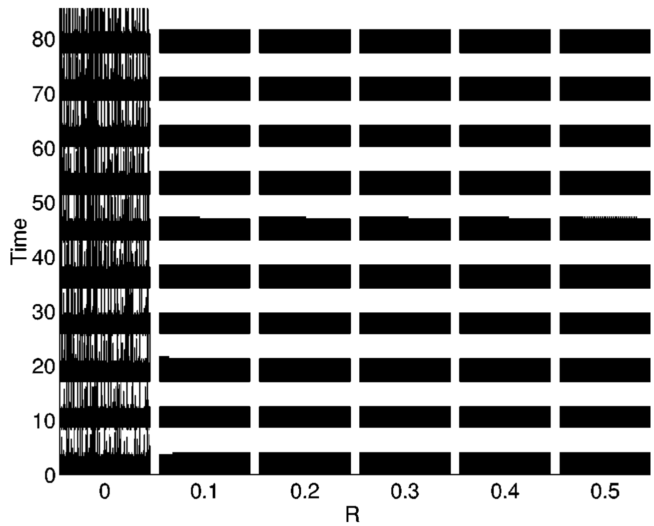

FIG. 5. Spatiotemporal synchronization of noisy nonlinear oscillators.

stability analysis. For the normal coupling, the SNR increment of $\epsilon>0.5/(\Delta t)$ is essentially the same as that of $\epsilon<0.5/(\Delta t)$ for any noise strength. This means that the normal coupling is still stable when $\epsilon>0.5/(\Delta t)$. Following the previous discussion, the stability analysis of the normal coupling might not need to be specifically conducted in this example. The normal coupling will be surely stable when the stable condition of the corresponding on-off process is satisfied, i.e., $\epsilon<0.5/(\Delta t)$.

The visual effect of spatiotemporal synchronization induced by the on-off coupling is depicted in Fig. 5. Each sequence in Fig. 5 displays the evolution of a chain of 101 oscillators, time increasing upward. Following Lindner *et al.* [9], binary signals obtained through quantization are shown. The strength of the white noise is chosen as -5 dB in this example. Obviously, strong spatiotemporal disorder appears before coupling (i.e., $R=0$), while excellent spatiotemporal synchronization is achieved by using the on-off coupling, for any employed R. The SNR improvement of unquantized signals is at least 26.

B. Application to information processing

Apart from nonlinear dynamical systems, the applications of the proposed controlled coupling process to IAR and DSP of real-world problems are also considered. One interesting IAR problem, the trend estimation of a benchmark time series, the "sales of company X" [15,18], is studied. Such a time series can be regarded as $U(t_1)$, produced by an unknown and unpredictable dynamic system, and its analysis is of practical importance. The sales is a monthly series ranging from January 1965 to May 1971 and has a monotonic growing trend and a clearly identifiable seasonal component. Guided by the earlier stability analysis, we choose three sets of (R,T) values, $(0.4,0)$, $(0,-0.12)$, and $(0.25,-0.05)$. For each set, the effect of changing the smoothing parameter S on the extracted trend is studied, i.e., the value of S varies from 1 to some large numbers, see Fig. 6. The Neumann boundary condition is used in this study. As expected, estimates with large smoothing parameters are very smooth, while for small values of S the estimates provide good interpolation of the data. This confirms previous theoretical study that the smoothing parameter S governs the smoothness-fidelity tradeoff for IAR. It is also clear from the figure that, with appropriate S, the estimates by using different R and T combinations are almost identical. The smoothing parameters used in Fig. 6(c) are nearly optimal. The corresponding estimated trends provide similar long-run tendency. Of particular importance is the slope of trends that undergoes a clear change around the 28th month, and agrees with the finding in Ref. [15]. For the study of the coupled nonlinear dynamics, the present numerical results indicate that the single next-to-the-nearest neighborhood coupling, Eq. (1), can reach the same effect as the single nearest coupling (i.e.,

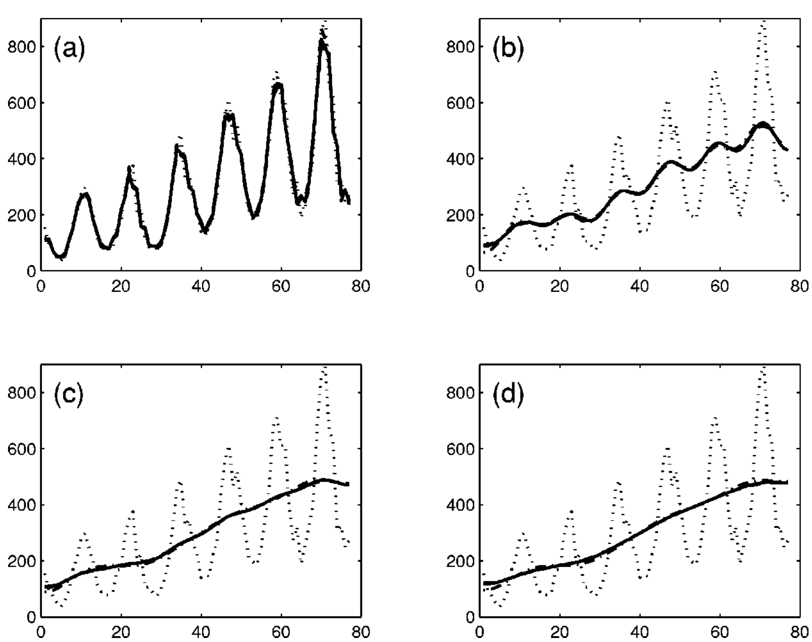

FIG. 6. Trend estimated by using the controlled coupling process as a function of S. Three combinations of R and T are tested, (1) $(0.4,0)$; (2) $(0,-0.12)$; (3) $(0.25,-0.05)$. In all plots, dotted lines denote the original data, while dashed, dash-dotted, and solid lines denote the trends estimated by using combinations (1), (2), and (3), respectively. (a) $S=1$ for all three combinations; (b) $S=20,250,27$; (c) $S=38,700,52$; (d) $S=60,1000,80$ for combinations (1), (2), and (3), respectively.

the normal coupling), and so does the combined coupling presented in Eq. (1). This agrees with the previous discussion.

C. Application to noise reduction

We finally consider a DSP problem, the signal extraction from noisy data, by the on-off coupling process (2). The underlying nonlinear dynamical system is chosen as the nonlinear Schrödinger equation [19]

$$i\frac{\partial \Psi}{\partial t}+\frac{\partial^2 \Psi}{\partial x^2}+2|\Psi|^2\Psi=0, \quad x\in[0,L], \tag{26}$$

with periodic boundary conditions $\Psi(x+L,t)=\Psi(x,t)$ and the period $L=2\sqrt{2}\pi$. This system is computationally difficult because of possible numerically induced chaos [19]. In this study, Eq. (26) is allowed to evolve freely with the initial condition of the form

$$\Psi(x,0)=0.5+0.05\cos\left(\frac{2\pi}{L}x\right)+i10^{-5}\sin\left(\frac{2\pi}{L}x\right). \tag{27}$$

At $t=t_1=8.0$, the system is perturbed and its solution is contaminated by the Gaussian white noise to the SNR of 39.25 dB, see Fig. 7. The controlled coupling process is used to restore the solution from noisy dynamical data $U(t_1)$ during the time period $t_1\leqslant t\leqslant t_2$. The parameters (R,T,S) are chosen as (0.25, -0.05, 10). From Fig. 7, it is clear that the unwanted noise is satisfactorily suppressed by the proposed process. The restored solution matches well with the noise-free solution and its SNR is as high as 50.63 dB.

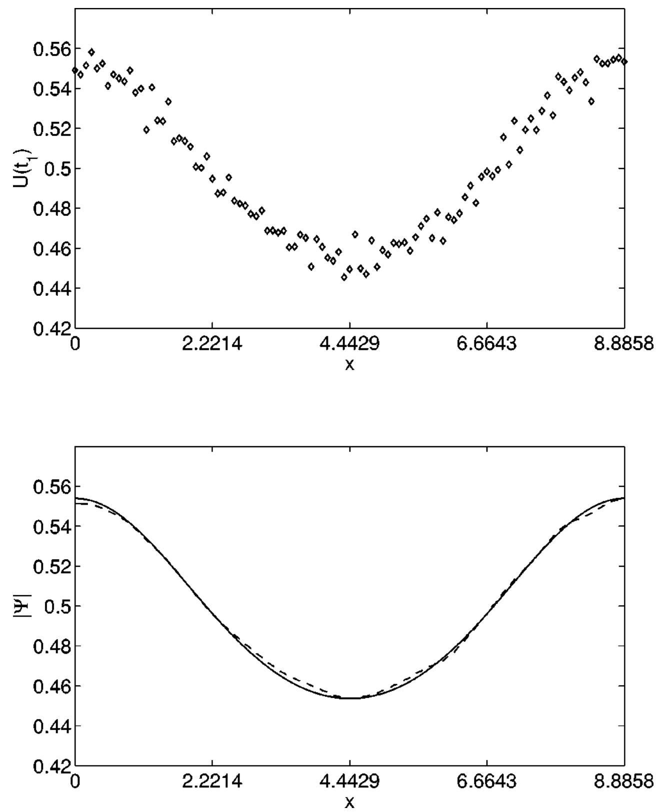

FIG. 7. Signal restoration from the contaminated solution of the nonlinear Schrödinger equation. Top, the contaminated solution $U(t_1)$; Bottom, the restored solution (dashed line) and the noise-free solution (solid line).

V. CONCLUSION

In conclusion, we introduce an on-off coupling process for the synchronization of spatiotemporal systems. The proposed process isolates the conventional coupling from the dynamical system and provides an in-depth analysis of the coupling effect. Numerical stability of the proposed process is analyzed, which might be useful for unisolated nonlinear dynamics. In the context of trend estimation, comparison is given to several standard nonparametric methods [13–15], which globally minimize the smoothness and fidelity. The proposed process is shown to balance these features in each step of time evolution, without resorting to a minimization procedure, and thus, is numerically simpler than the existing methods. The IAR properties of the coupling are shown to be comprehensible from the viewpoint of nonlinear dynamical studies. In the context of signal processing, a digital filter expression of the controlled coupling process is derived and the connection of the proposed approach to the standard Hanning filter [17] is made. For the study of nonlinear dynamical system, the present discussion provides a DSP explanation about why the coupling induces synchronization. On the other hand, the on-off process also forms an interesting means to determine whether a new neighborhood coupling will introduce synchronization or not. The proposed process is finally applied to chaos synchronization, spatiotemporal synchronization, trend estimation, and signal restoration.

Essentially, a coupling process is a low-pass filter in general and therefore, it eliminates the high-frequency component, the difference among the dynamical variables of nonlinear systems and leads to synchronization. The conventional coupling and the proposed on-off coupling are two different implementations of low-pass filters. The former is an interactive implementation and the latter is a postprocessing implementation. Both approaches work equally well for chaos synchronization of nonlinear dynamical systems.

In the numerical experiment of chaos synchronization, by using the controlled coupling, the synchronous state can be reached much more quickly than by using the conventional coupling. In the spatiotemporal synchronization, excellent spatiotemporal order can be induced by using the proposed process, and the overall SNR improvement is comparable to the conventional coupling. The proposed controlled coupling process is also applied to information processing problems. A benchmark time series generated by some unknown dynamical process is studied. The smoothness-fidelity balancing via smoothing parameters is numerically verified. The similar and excellent trends are estimated by using three different sets of parameters. The numerical results are in good agreement with those in the literature [15] and the present approach is simpler. Finally, the signal restoration is studied. The solution of the nonlinear Schrödinger equation is contaminated by noise at the end of the first time period t_1. The controlled coupling process is utilized to restore the wave form. We show that the unwanted noise can be effectively removed by ten iterations. Obviously, the proposed approach is readily applicable to other real-world information processing problems, such as image processing. The present investigation opens up a new opportunity to develop other synchronization-based methods both for the study of nonlinear dynamics and for realistic information processing.

ACKNOWLEDGMENTS

This work was supported in part by the National University of Singapore.

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